JEE MAINS-2017

08-04-2017 (Online)

IMPORTANT INSTRUCTIONS

1. The test is of **3** hours duration.
2. The Test Booklet consists of **90** questions. The maximum marks are **360**.
3. There are **three** parts in the question paper A, B, C consisting of **Mathematic, Physics & Chemistry** having 30 questions in each part of equal weightage. Each question is allotted **4 (four)** marks for each correct response.
4. Candidates will be awarded marks as stated above in instruction No.3 for correct response of each question. 1/4 (one fourth) marks will be deducted for indicating incorrect response of each question. No deduction from the total score will be made if no response is indicated for an item in the answer sheet.
5. There is only one correct response for each question. Filling up more than one response in each question will be treated as wrong response and marks for wrong response will be deducted accordingly as per instruction 4 above.

PART - A - MATHEMATIS

1. Let f (x) = $2^{10} \cdot x + 1$ and g(x) = $3^{10} \cdot x - 1$. If (fog)(x) = x, then x is equal to

(A) $\frac{3^{10}-1}{3^{10}-2^{-10}}$ (B) $\frac{2^{10}-1}{2^{10}-3^{-10}}$ (C) $\frac{1-3^{-10}}{2^{10}-3^{-10}}$ (D*) $\frac{1-2^{-10}}{3^{10}-2^{-10}}$

Sol. f(g(x)) = x

$f(3^{10}x - 1) = 2^{10}(3^{10}.x - 1) = x$

$= \frac{1}{3^{10}-2^{-110}}$

$2^{10}(3^{10}x - 1) + 1 = x$

$x(6^{10} - 1) = 2^{10} - 1$

$x = \frac{2^{10}-1}{6^{10}-1} \qquad = \frac{1-2^{-10}}{3^{10}-2^{-10}}$

2. Let p(x) be a quadratic polynomial such that p(0) = 1. If p(x) leaves remainder 4 when divided by x – 1 and it leaves remainder 6 when divided by x + 1; then

(A) p(2) = 11 (B) p(2) = 19 (C*) p(– 2) = 19 (D) p(– 2) = 11

Sol. $p(x) = ax^2 + bx + c$

p (0) = 1 = c = 1

$\left.\begin{matrix} p(1) = 4 \\ p(-1) = 6 \end{matrix}\right]$

$\left.\begin{matrix} a+b+c = 4 \\ a-b+c = 6 \end{matrix}\right] = \left.\begin{matrix} a = 4 \\ b = -1 \end{matrix}\right]$

$p(x) = 4x^2 - x + 1$

p(–2) = 16 + 2 + 1 = 19

3. Let z ∈ C, the set of complex numbers. Then the equation, 2 |z + 3i | – |z – i| = 0 represents:

(A*) a circle with radius $\frac{8}{3}$ (B) a circle with diameter $\frac{10}{3}$

(C) an ellipse with length of major axis $\frac{16}{3}$ (D) an ellipse with length of minor axis $\frac{16}{9}$

Sol. 2 |x + i (y + 3)| = |x + i (y – 1)|

$= 2\sqrt{x^2+(y+3)^2} \qquad \sqrt{x^2+(y-1)^2}$

$= 4x^2 + 4(y+3)^2 = x^2 + (y-1)^2$

$= 3x^2 = y^2 - 2y + 1 - 4y^2 - 24y - 36$

$= 3x^2 + 3y2 + 26y + 35 = 0$

$= x^2 + y^2 + \frac{26}{3}y + \frac{35}{3} = 0$

$= r = \sqrt{0 + \frac{169}{9} - \frac{35}{3}}$

$= \sqrt{\frac{64}{9}} = \frac{8}{3}$

4. The number of real values of λ for which the system of linear equations

$2x + 4y - \lambda z = 0$

$4x + \lambda y + 2z = 0$

$\lambda x + 2y + 2z = 0$

has infinitely many solutions, is

(A) 0 (B*) 1 (C) 2 (D) 3

Sol. $\Delta = 0$

$$\begin{vmatrix} 2 & 4 & -\lambda \\ 4 & \lambda & z \\ \lambda & z & 2 \end{vmatrix} \begin{matrix} 2 & 4 \\ 4 & \lambda \\ \lambda & z \end{matrix} = 0$$

$= -(32 + 8 - \lambda^3) = 0$

$= \lambda^3 + 4\lambda - 40 = 0$

5. Let A be any 3× 3 invertible matrix. Then which one of the following is not always true?

(A) adj (A) = $|A| \cdot A^{-1}$

(B) adj (adj(A)) = $|A| \cdot A$

(C) adj (adj(A)) = $|A|^2 \cdot (\text{adj}(A))^{-1}$

(D*) adj (adj(A)) = $|A| \cdot (\text{adj}(A))^{-1}$

Sol. See theory

6. If all the words, with or without meaning, are written using the letters of the word QUEEN and are arranged as in English dictionary, then the position of the word QUEEN is

(A) 44^{th} (B) 45^{th} (C*) 46^{th} (D) 47^{th}

Sol. E,E, N, Q,U

(i) E......... = 24

(ii) N .... $= \frac{4!}{2} = 12$

(iii) Q E........ = 3! = 6

(iv) Q N ...... $= \frac{3!}{2!} = 3$

(v) Q U E E N = 1

Total = (i) + (ii) + (iii) + (iv) + (v) = 46^{th}

7. If $(27)^{999}$ is divided by 7, then the remainder is :

(A) 1 (B) 2 (C) 3 (D*) 6

Sol. $\frac{(28-1)^{999}}{7} = \frac{28\lambda - 1}{7} \Rightarrow \frac{28\lambda - 7 + 1 - 1}{7} = \frac{7(4\lambda - 1) + 6}{7}$

$\therefore$ Rem = 6

8. If the arithmetic mean of two numbers a and b, a > b > 0, is five times their geometric mean, then $\frac{a+b}{a-b}$ is equal to

(A) $\frac{\sqrt{6}}{2}$ (B) $\frac{3\sqrt{2}}{4}$ (C) $\frac{7\sqrt{3}}{12}$ (D*) $\frac{5\sqrt{6}}{12}$

Sol. $\frac{a+b}{2} = 5\sqrt{ab}$

$\frac{a+b}{\sqrt{ab}} = 10$

$\therefore \frac{a}{b} = \frac{10+\sqrt{96}}{10-\sqrt{96}} = \frac{10+4\sqrt{6}}{10-4\sqrt{6}}$

Use C and D

$\frac{a+b}{a-b} = \frac{20}{8\sqrt{6}} = \frac{5}{2\sqrt{6}} = \frac{5\sqrt{6}}{12}$

9. If the sum of the first n terms of the series $\sqrt{3}+\sqrt{75}+\sqrt{243}+\sqrt{507}+$........is $435\sqrt{3}$ then n equals

(A) 18 (B*) 15 (C) 13 (D) 29

Sol. $\sqrt{3}\left[1+\sqrt{25}+\sqrt{81}+\sqrt{169}+......\right] = 435\sqrt{3}$

$\sqrt{3}\left[1+5+9+13+.....T_n\right] = 435\sqrt{3}$

$= \sqrt{3}\times\frac{n}{2}\left[2+(n-1)4\right] = 435\sqrt{3}$

$2n + 4n^2 - 4n = 870$

$= 4n^2 - 2n - 870 = 0$

$= 2n^2 - n - 435 = 0$

$n = \frac{1\pm\sqrt{1+4\times2\times435}}{4}$

$= \frac{1\pm 59}{4}$

$= \frac{1+59}{4} = 4; \frac{1-59}{4}$

10. $\lim_{x\to3}\frac{\sqrt{3x}-3}{\sqrt{2x-4}-\sqrt{2}}$ is equal to

(A) $\sqrt{3}$ (B*) $\frac{1}{\sqrt{2}}$ (C) $\frac{\sqrt{3}}{2}$ (D) $\frac{1}{2\sqrt{2}}$

Sol. $\lim\limits_{x\to\infty}\dfrac{\sqrt{3x}-3}{\sqrt{2x-4}-\sqrt{2}}$

Rationalize

$$\lim_{x\to\infty}\frac{(3x-9)\times\left(\sqrt{3x-4}-\sqrt{2}\right)}{\{(2x-4)\}\times\left(\sqrt{3x}+3\right)}$$

$$=\lim_{x\to\infty}\frac{3(x-3)}{2(x-3)}\times\frac{\sqrt{2x-4}+\sqrt{2}}{\left(\sqrt{3x}+3\right)}$$

$$=\frac{3}{2}\times\frac{2\sqrt{2}}{6}=\frac{1}{\sqrt{2}}$$

11. The tangent at the point (2, – 2) to the curve, $x^2y^2 - 2x = 4(1-y)$ does not pass through the point

(A) $\left(4,\dfrac{1}{3}\right)$ (B) (8, 5) (C) (–4, – 9) (D*) (–2, – 7)

Sol. $x^2y^2 - 2x = 4 - 4y$

$$2xy^2+2y.x^2.\frac{dy}{dx}-2=-4.\frac{dy}{dx}$$

$$\frac{dy}{dx}\left(2y.x^2+4\right)=2-2x.y^2$$

$$\left.\frac{dy}{dx}\right|_{2,-2}=\frac{2-2\times2\times4}{2(-2)\times4+4}=\frac{+14}{+12}=\frac{7}{6}$$

$$(y+2)=\frac{7}{6}(x-2)\Rightarrow 7x-6y=26$$

12. If $y=\left[x+\sqrt{x^2-1}\right]^{15}+\left[x-\sqrt{x^2-1}\right]^{15}$, then $(x^2-1)\dfrac{d^2y}{dx^2}+x\dfrac{dy}{dx}$ is equal to

(A) 125y (B) 224 y^2 (C) 225 y^2 (D*) 225 y

Sol. $y=\left\{x+\sqrt{x^2-1}\right\}^{15}+\left\{x-\sqrt{x^2-1}\right\}^{15}$

$$\frac{dy}{dx}=15\left\{x+\sqrt{x^2-1}\right\}^{14}15\left\{x-\sqrt{x^2-1}\right\}^{14}\left(1-\frac{x}{\sqrt{x^2-1}}\right)$$

$$\frac{dy}{dx}=\frac{15}{\sqrt{x^2-1}}.y \qquad ...(i)$$

$$\sqrt{x^2-1}.\frac{dy}{dx}=15y$$

$$\frac{x}{\sqrt{x^2-1}}.\frac{dy}{dx}+\sqrt{x^2-1}\frac{d^2y}{dx^2}=15\frac{dy}{dx}$$

$$x\frac{dy}{dx}+\left(x^2-1\right)\frac{d^2y}{dx^2}=15\sqrt{x^2-1}.\frac{15}{\sqrt{x^2-1}}.y=225y$$

13. If a point P has co-ordinates (0, – 2) and Q is any point on the circle, $x^2 + y^2 - 5x - y + 5 = 0$, then the maximum value of $(PQ)^2$ is

(A) $\frac{25+\sqrt{6}}{2}$ (B*) $14+5\sqrt{3}$ (C) $\frac{47+10\sqrt{6}}{2}$ (D) $8+5\sqrt{3}$

Sol. $(x-5/2)^2 \frac{-25}{4} + (y-1/2)^2 - 1/4 + 5 = 0$

$= (x-5/2)^2 + (y-1/2)^2 = 3/2$

on circle Q = 5/2 + 3/ 2 cos Q, $\frac{1}{2}+\sqrt{3/2}\sin Q$

$$PQ^2 = \left(\frac{5}{2}+\sqrt{3/2}\cos Q\right)^2 + \left(\frac{5}{2}+\sqrt{3/2}\sin Q\right)^2$$

$$PQ^2 = \frac{25}{2}+\frac{3}{2}+5\sqrt{3/2}(\cos Q+\sin Q)$$

$$= 14+5\sqrt{3/2}(\cos Q+\sin Q)$$

$$\text{man}^{mr} = 14+5\sqrt{3/2}(\sqrt{2})$$

$$= 14+5\sqrt{3}$$

14. The integral $\int\sqrt{1+2\cot x(\operatorname{cosec} x+\cot x)}\,dx \left(0<x<\frac{\pi}{2}\right)$ is equal to

(where C is a constant of integration)

(A) $4\log\left(\sin\frac{x}{2}\right)+C$ (B*) $2\log\left(\sin\frac{x}{2}\right)+C$

(C) $2\log\left(\cos\frac{x}{2}\right)+C$ (D) $4\log\left(\cos\frac{x}{2}\right)+C$

Sol. $\int\left(\sqrt{+2\cot x\ \text{coesc}\,x+\operatorname{cosec}^2 x+\cot x}\right)dx$

∫cos | x + cot x | dx

∫(cosec + cot x)dx

∫cosec dx

$2\log(\log(x_2)) + c$

15. The integral $\int_{\frac{\pi}{12}}^{\frac{\pi}{4}} \frac{8\cos 2x}{(\tan x+\cot x)^3}dx$ equals

(A*) $\frac{15}{128}$ (B) $\frac{15}{64}$ (C) $\frac{13}{32}$ (D) $\frac{13}{256}$

Sol. $$\int_{\frac{\pi}{12}}^{\frac{\pi}{4}} \frac{\cos 2x}{\left(\frac{1}{\sin 2x}\right)^3} = \int_{\frac{\pi}{12}}^{\frac{\pi}{4}} \cos 2x \times \sin 2x \sin^2(2x)dx$$

$$= \frac{1}{4}\int \sin 4x.(1-\cos 4x)dx$$

$$= \frac{1}{4}\int_{\frac{\pi}{12}}^{\frac{\pi}{4}} \sin 4x - \int_{\frac{\pi}{12}}^{\frac{\pi}{4}} \sin 8x$$

16. The area (in sq. units) of the smaller portion enclosed between the curves, $x^2 + y^2 = 4$ and $y^2 = 3x$, is

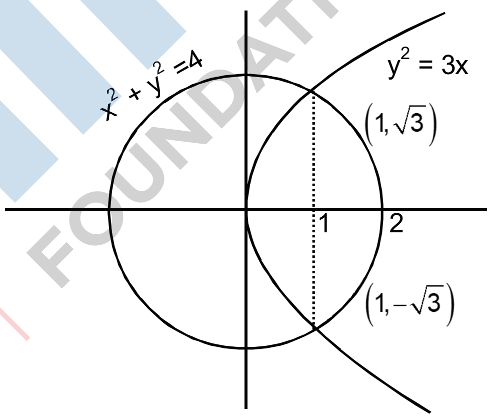
(A) $\frac{1}{2\sqrt{3}} + \frac{\pi}{3}$ (B) $\frac{1}{\sqrt{3}} + \frac{2\pi}{3}$ (C) $\frac{1}{2\sqrt{3}} + \frac{2\pi}{3}$ (D*) $\frac{1}{\sqrt{3}} + \frac{4\pi}{3}$

Sol. $x^2 + 3x - 4 = 0$

$(x + 4)(x - 1) = 0$

$x = -4, x = 1$

$$\text{Area} = \left(\int_0^1 \sqrt{3}.\sqrt{x}\right) + \int_0^1\left(\left(\int_0^1 \sqrt{3}.\sqrt{x}dx + \int_1^2 \sqrt{4-x^2}.dx\right)\right) \times 2$$



$$= \left(\sqrt{3}\left(\frac{x^{3/2}}{3/2}\right)_0^1 + \left(\frac{x}{2}\sqrt{4-x^2} + 2\sin^{-1}\frac{x}{2}\right)_1^2\right) \times 2$$

$$= \left(\sqrt{3}\left(\frac{2}{3}\right) + \left\{2.\frac{\pi}{2} - \left(\frac{\sqrt{3}}{2} + \frac{\pi}{2}\right)\right\}\right) \times 2$$

$$\left(\frac{2}{\sqrt{3}} - \frac{\sqrt{3}}{2} + \frac{2\pi}{3}\right) \times 2$$

$$= \left(\frac{1}{2\sqrt{3}} + \frac{2\pi}{3}\right) \times 2 = \frac{1}{\sqrt{3}} + \frac{4\pi}{3}$$

17. The curve satisfying the differential equation, $y\,dx - (x + 3y^2)dy = 0$ and passing through the point (1, 1), also passes through the point

(A) $\left(\frac{1}{4}, \frac{-1}{2}\right)$ (B*) $\left(\frac{-1}{3}, \frac{1}{3}\right)$ (C) $\left(\frac{1}{3}, \frac{-1}{3}\right)$ (D) $\left(\frac{1}{4}, \frac{1}{2}\right)$

Sol. $dx - xdy - 3y^2\,dy = 0$

$$\frac{dx}{dy} = \frac{x}{y} + 3y$$

$$\frac{dx}{dy} - \frac{x}{y} = 3y$$

$$\text{I.f.} = e^{-\int \frac{1}{y}dy} = e^{-\ln y} = \frac{1}{y}$$

$\therefore$ solution is

$$\frac{x}{y} = \int 3y.\frac{1}{y}dy$$

$$\frac{x}{y} = 3y + c$$

pass through (1,1) $\therefore$ 1 = 3 + c ; c = –20

$x = 3y^2 – 2y$

(i) $\left(\frac{1}{4}, -\frac{1}{2}\right) = \frac{1}{4} = \frac{3}{4} + 1$

(ii) $-\frac{1}{3} = \frac{1}{3} - \frac{2}{3} = -\frac{1}{3}$

18. The locus of the point of intersection of the straight lines, tx – 2y – 3t = 0, x –2ty + 3 = 0 ($t \in R$), is

(A) an ellipse with eccentricity $\frac{2}{\sqrt{5}}$

(B) an ellipse with the length of major axis 6

(C) a hyperbola with eccentricity $\sqrt{5}$

(D*) a hyperbola with the length of conjugate axis 3

Sol. tx – 2y – 3t = 0

x – 2ty + 3 = 0

tx – 2y – 3t = 0 $t^2x – 2ty – 3t^2 = 0$

$tx – 2t^2y + 3t = 0$ x – 2ty + 3 = 0

\+   –   –   +   –

$y(2t^2 –2) = 6t$ $(t^2 -1)x = (3t^2 + 1)$

$y = \frac{6t}{2t^2 - 2} = \frac{3t}{t^2 - 1}$ $x = – 3\sec 2\theta$

$2y = 3(–\tan 2\theta)$

$\sec^2 2\theta – \tan^2 2\theta = 1$

$$\frac{x^2}{9} - \frac{y^2}{9/4} = 1$$

$a^2 = 9$; $b^2 = 9/4$

λ(T.A) = 6 ; $e^2 = 1 + \frac{9/4}{9} = 1 + \frac{1}{4}; e = \frac{\sqrt{5}}{2}$

19. If two parallel chords of a circle, having diameter 4 units, lie on the opposite sides of the centre and subtend angles $\cos^{-1}\left(\frac{1}{7}\right)$ and $\sec^{-1}(7)$ at the centre respectively, then the distance between these chords, is

(A) $\frac{4}{\sqrt{7}}$ (B*) $\frac{8}{\sqrt{7}}$ (C) $\frac{8}{7}$ (D) $\frac{16}{7}$

Sol. cos 2Q = 1/7 = $2\cos^2Q - 1 = 1/7$

$= 2\cos^2Q = 8/7$

$\cos^2Q = 4/7$

$= \frac{cp^2}{4} = \frac{4}{7}$

$= Cp = \frac{4}{\sqrt{7}}$

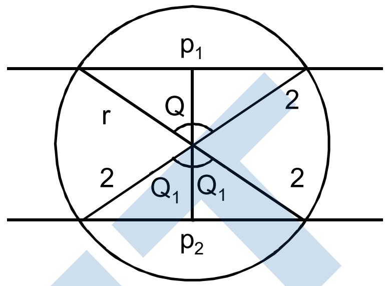
sec2Q = 7 $= \frac{1}{2\cos^2 Q - 15} = 7$

$= 2\left(\frac{Cp_2}{2}\right)^2 - 1 = \frac{1}{7}$

$= 2\left(\frac{Cp_2}{2}\right)^2 = \frac{8}{7}$

$= \boxed{Cp_2 = \frac{4}{\sqrt{7}}}$

$\frac{4}{\sqrt{7}} + \frac{4}{\sqrt{7}} = \frac{8}{\sqrt{7}}$



20. If the common tangents to the parabola, $x^2 = 4y$ and the circle, $x^2 + y^2 = 4$ intersect at the point P, then the distance of P from the origin, is

(A) $\sqrt{2} + 1$ (B) $2(3 + 2\sqrt{2})$ (C*) $2(\sqrt{2} + 1)$ (D) $3 + 2\sqrt{2}$

Sol. tangent to $x^2 + y^2 = 4$

$y = mx \pm 2\sqrt{1+m^2}$

$x^2 = 4y$

$x^2 = 4mx + 8\sqrt{1+m^2}$

$x^2 = 4mx - 8\sqrt{1+m^2} = 0$

D = 0

$16m^2 + 4.8\sqrt{1+m^2} = 0$

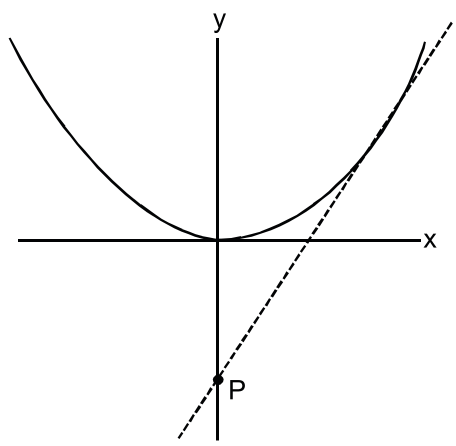
$m^2 + 2\sqrt{1+m^2} = 0$

or $m^2 = \sqrt{1+m^2}$

$m^4 = 4 + 4m^2$

$m^4 - 4m^2 - 4 = 0$

$m^2 = \frac{4 \pm \sqrt{16+16}}{2}$



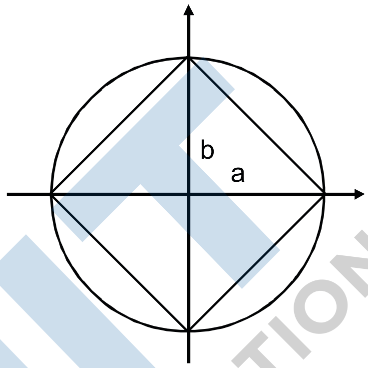
$$= \frac{4 \pm 4\sqrt{2}}{2}$$

$$m^2 = 2 + 2\sqrt{2}$$

21. Consider an ellipse, whose centre is at the origin and its major axis is along the x-axis. If its eccentricity is $\frac{3}{5}$ and the distance between its foci is 6, then the area (in sq. units) of the quadrilateral inscribed in the ellipse, with the vertices as the vertices of the ellipse, is

(A) 8 (B) 32 (C) 80 (D*) 40

Sol.



e = 3/5, 2ae = 6, a(5) a = 5

$b^2 = a^2 (1–e^2)$

$b^2 = 25 (1–9/25)$

b = 4

area = 4 (1/2 ab)

= 2ab = 40

22. The coordinates of the foot of the perpendicular from the point (1, – 2, 1) on the plane containing the lines, $\frac{x+1}{6} = \frac{y-1}{7} = \frac{z-3}{8}$ and $\frac{x-1}{3} = \frac{y-2}{5} = \frac{z-3}{7}$ is

(A) (2, – 4, 2) (B) (–1, 2, – 1) (C*) (0, 0, 0) (D) (1, 1, 1)

Sol. $\bar{n} = \bar{n}_1 \times \bar{n}_2$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 6 & 7 & 8 \\ 3 & 5 & 7 \end{vmatrix}$$

= (9, –18, 9)

= (1, –2, 1)

1(x+1)–2 (y–1) + (2–3) = 0

$= \boxed{x - 2y + z = 0}$

foot to z

$$\frac{x-1}{1} = \frac{y+2}{-2} = \frac{z-1}{1} = -\frac{[1+4+1]}{6}$$

$$\boxed{x = 0, y = 0, z = 0}$$

23. The line of intersection of the planes $\vec{r} \cdot (3\hat{i} - \hat{j} + \hat{k}) = 1$ and $\vec{r} \cdot (\hat{i} + 4\hat{j} - 2\hat{k}) = 2$, is

(A) $\frac{x - \frac{4}{7}}{-2} = \frac{y}{7} = \frac{z - \frac{5}{7}}{13}$

(B) $\frac{x - \frac{4}{7}}{2} = \frac{y}{-7} = \frac{z + \frac{5}{7}}{13}$

(C*) $\frac{x - \frac{6}{13}}{2} = \frac{y - \frac{5}{13}}{-7} = \frac{z}{-13}$

(D) $\frac{x - \frac{6}{13}}{2} = \frac{y - \frac{5}{13}}{7} = \frac{z}{-13}$

Sol. $\bar{n} = \bar{n}_1 \times \bar{n}_2$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -1 & 1 \\ 1 & 4 & -2 \end{vmatrix}$$

$= \hat{i}(-2) - \hat{j}(-7) + \hat{k}(13)$

$\bar{n} = 2\hat{i} + 7\hat{j} + 13\hat{k}$

Now

3x – y + z = 1

x + 4y – 2z = 2

but z = 0

3x – y = 1×4

x + 4y = 2

13x = 6 x = 6/13

y = 5/13

.... is

$$\boxed{\frac{x-6/13}{-2} = \frac{y-5/13}{7} = \frac{z-0}{13}}$$

or

$$\boxed{\frac{x-6/13}{2} = \frac{y-5/13}{-7} = \frac{z}{-13}}$$

24. The area (in sq. units) of the parallelogram whose diagonals are along the vectors $8\hat{i} - 6\hat{j}$ and $3\hat{i} + 4\hat{j} - 12\hat{k}$, is

(A) 26 (B*) 65 (C) 20 (D) 52

Sol. $d_1 \times d_2 = \begin{vmatrix} I & J & K \\ 8 & -6 & 0 \\ 3 & 4 & -12 \end{vmatrix}$

$= 72\hat{i} - (-96)\hat{j} + 50\hat{k}$

= 5178 + 7056 + 2500

$|d_1 \times d_2| = \sqrt{16900} = 130$

$A = \frac{1}{2}|d_1 \times d_2| = \frac{1}{2} \times 130$

= 65

25. The mean age of 25 teachers in a school is 40 years. A teacher retires at the age of 60 years and a new teacher is appointed in his place. If now the mean age of the teachers in this school is 39 years, then the age (in years) of the newly appointed teacher is :

(A) 25 (B) 30 (C*) 35 (D) 40

Sol. $\frac{x_1 + x_2 + \ldots + x_{25}}{25} = \bar{x} = 40$

$x_1 + x_2 + \ldots + x_{25} = 1000$

$x_2 + x_2 + \ldots + x_{25-60} + A = \bar{x} \times 25$

$1000 - 60 + A = 39 \times 25 = 975$

$A = 975 - 940 = 35$

26. Three persons P, Q and R independently try to hit a target. If the probabilities of their hitting the target are $\frac{3}{4}, \frac{1}{2}$ and $\frac{5}{8}$ respectively, then the probability that the target is hit by P or Q but not by R is

(A*) $\frac{21}{64}$ (B) $\frac{9}{64}$ (C) $\frac{15}{64}$ (D) $\frac{39}{64}$

Sol. $\left(\frac{3}{4}\right)\left(\frac{1}{2}\right)\left(\frac{3}{8}\right) + \left(\frac{1}{4}\right)\left(\frac{1}{2}\right)\left(\frac{3}{8}\right) + \left(\frac{3}{4}\right)\left(\frac{1}{2}\right)\left(\frac{3}{8}\right)$

$= \frac{12+9}{64}$

$= \frac{27}{64}$

27. An unbiased coin is tossed eight times. The probability of obtaining at least one head and at least one tail is

(A) $\frac{255}{256}$ (B*) $\frac{127}{128}$ (C) $\frac{63}{64}$ (D) $\frac{1}{2}$

Sol. 1 – ∫ P(All Head) + P(AllTail)

$1 - \left\{\frac{1}{2^8} + \frac{1}{2^8}\right\}$

$= 1 - \frac{1}{2^7}$

$= 1 - \frac{1}{128} = \frac{127}{128}$

28. If S = $\left\{x \in [0, 2\pi] : \begin{vmatrix} 0 & \cos x & -\sin x \\ \sin x & 0 & \cos x \\ \cos x & \sin x & 0 \end{vmatrix} = 0\right\}$, then $\sum_{x \in S} \tan\left(\frac{\pi}{3} + x\right)$ is equal to

(A) $4 + 2\sqrt{3}$ (B) $-2 + \sqrt{3}$ (C) $-2 - \sqrt{3}$ (D*) $-4 - 2\sqrt{3}$

Sol. $0(0 - \cos x) - \cos x(0 - \cos^2 x) - \sin x(\sin^2 x - 0) = 0$

$\cos^3 x - \sin^3 x = 0$

$\tan^3 = 1 \Rightarrow \tan x = 1$

$$\sum \frac{\sqrt{3}+\tan x}{1-\sqrt{3}}$$

$$\sum \frac{\sqrt{3}+1}{1-\sqrt{3}} \times \frac{\sqrt{3}+1}{1-\sqrt{3}} \Rightarrow \sum \frac{1+3+2\sqrt{3}}{-2} = \sum \frac{4^2}{-2} - \frac{2\sqrt{3}}{3}$$

$$\sum -2-\sqrt{3}$$

29. The value of $\tan^{-1}\left[\frac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}-\sqrt{1-x^2}}\right]$, $|x| < \frac{1}{2}$, $x \neq 0$, is equal to

(A*) $\frac{\pi}{4} + \frac{1}{2}\cos^{-1} x^2$ (B) $\frac{\pi}{4} + \cos^{-1} x^2$ (C) $\frac{\pi}{4} - \frac{1}{2}\cos^{-1} x^2$ (D) $\frac{\pi}{4} - \cos^{-1} x^2$

Sol. $x^2 = \cos 2\theta$; $\theta = \frac{1}{2}\cos x^2$

$$\tan^{-1}\left[\frac{\sqrt{1+\cos 2\theta}+\sqrt{1-\cos 2\theta}}{\sqrt{1+\cos 2\theta}-\sqrt{1-\cos 2\theta}}\right]$$

$$\tan^{-1}\left[\frac{1+\tan\theta}{1-\tan\theta}\right]$$

$$= \tan^{-1}\left[\tan\left(\frac{\pi}{4}+\theta\right)\right]$$

$$= \frac{\pi}{2} + \frac{1}{2}\cos x^2$$

30. The proposition $(\sim p) \vee (p \wedge \sim q)$ is equivalent to

(A) $p \vee \sim q$ (B*) $p \rightarrow \sim q$ (C) $p \wedge \sim q$ (D) $q \rightarrow p$

Sol. (~ P) v (p^ ~ q)

p	q	~ p	~ q	p^ ~ q	(~ p)v(p^ ~ q)
T	T	F	F	F	F
T	F	F	T	T	T
F	T	T	F	F	F
F	F	T	T	F	F

PART – B – PHYSICS

31. A compressive force, F is applied at the two ends of a long thin steel rod. It is heated, simultaneously, such that its temperature increases by ΔT. The net change in its length is zero. Let ℓ be the length of the rod, A its area of cross-section, Y its Young's modulus, and α its coefficient of linear expansion. Then, F is equal to -

(A) $\ell A\, Y\alpha\, \Delta T$ (B) $A\, Y\alpha\, \Delta T$ (C*) $\frac{Ay}{\alpha \Delta T}$ (D) $\ell^2\, Y\alpha\, \Delta T$

Sol. Net change in length = 0

Thermal Exp. = $\ell \propto \Delta t$

$$y = \frac{F/A}{\Delta/\ell}$$

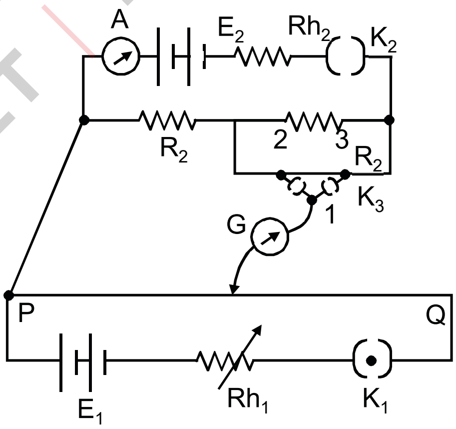
$$\frac{\Delta\ell}{\ell} = \frac{F}{Ay}$$

$$\Delta\ell = \frac{F\ell}{Ay}$$

$$\frac{F\ell}{Ay} = \ell \propto \Delta t$$

F = Ay $\propto$ Δt

32. A potentiometer PQ is set up to compare two resistances as shown in the figure. The ammeter A in the circuit reads 1.0 A when two way key K_3 is open. The balance point is at a length ℓ_1 cm from P when two way key K_3 is plugged in between 2 and 1, while the balance point is at a length ℓ_2 cm from P when key K_3 is plugged in between 3 and 1. The ratio of the two resistance $\frac{R_1}{R_2}$, is found to be -



(A*) $\frac{\ell_1}{\ell_1 - \ell_2}$ (B) $\frac{\ell_2}{\ell_2 - \ell_1}$ (C) $\frac{\ell_1}{\ell_1 + \ell_2}$ (D) $\frac{\ell_1}{\ell_2 - \ell_1}$

Sol. When key is at point

$V_1 = iR_1 = x\ell_1$

when key is at (3)

$V_2 = i\,(R_1 + R_2) = xl_2$

$$\frac{R_1}{R_1 + R_2} = \frac{\ell_1}{\ell_2}$$

$$\frac{R_1}{R_2} = \frac{\ell_1}{\ell_2 - \ell_1}$$

33. A signal of frequency 20 kHz and peak voltage of 5 volt is used to modulate a carrier wave of frequency 1.2 MHz and peak voltage 25 volts. Choose the correct statement.

(A) Modulation index = 5, side frequency bands are at 1400 kHz and 1000 kHz

(B) Modulation index = 0.8, side frequency bands are at 1180 kHz and 1220 kHz

(C*) Modulation index = 0.2, side frequency bands are at 1200 kHz and 1180 kHz

(D) Modulation index = 5, side frequency bands are at 21/2 kHz and 18.8 kHz

Sol. Modulation idex = m $= \frac{V_m}{V_0}$

$$= \frac{1}{5} = 0.2$$

Frequency = 12×10^3 kHz

F = 12.00 kHz

$F_1 = 1200 - 20 = 1180$ kHz

$F_2 = 1200 + 20 = 1220$ kHz

34. A single slit of width b is illuminated by a coherent monochromatic light of wavelength λ. If the second and fourth minima in the diffraction pattern at a distance 1 m from the slit are at 3 cm and 6 cm respectively from the central maximum, what is the width of the central maximum ? (i.e., distance between first minimum on either side of the central maximum)

(A) 4.5 cm (B) 1.5 cm (C) 6.0 cm (D*) 3.0 cm

Sol. min.

f sin $\theta = n\lambda$

$$\sin\theta = \frac{n\lambda}{6}$$

n = 2

$$\sin\theta = \frac{2\lambda}{6} = \tan\theta_1 = \frac{x_1}{D}$$

x = 4

$$\sin\theta_2 = \frac{4\lambda}{6} = \frac{x_2}{D}$$

$$x_2 - x_1 = \frac{4\lambda}{6} - \frac{2\lambda}{6} = \frac{2\lambda}{6} = 3 \text{ cm}$$

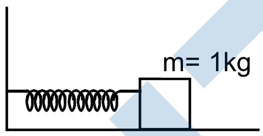
width of central max $= \frac{2\lambda}{6} = 3$ cm

35. A 1 kg block attached to a spring vibrates with a frequency of 1 Hz on a frictionless horizontal table. Two springs identical to the original spring are attached in parallel to an 8 kg block placed on the same table. So, the frequency of vibration of the 8 kg block is -

(A) 2 Hz (B) $\frac{1}{4}$Hz (C) $\frac{1}{2\sqrt{2}}$Hz (D*) $\frac{1}{2}$Hz

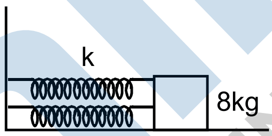
Sol. $F = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = 1$

$4\pi^2 = \frac{k}{m}$ $m = 1$



$k = 4\pi^2$

In parallel $k_{eq} = 2k$



$F = \frac{1}{2\pi}\sqrt{\frac{k \times 2}{8}}$

$= \frac{1}{2}$Hz

36. A magentic dipole in a constant magnetic field has -

(A) maximum potential energy when the torque is maximum

(B*) zero potential energy when the torque is maximum

(C) zero potential energy when the torque is minimum

(D) minimum potential energy when the torque is maximum

Sol. PE = – PE cos θ

τ = PE sin θ

τ_{max} when $\theta = 90°$

PE = 0

37. If the earth has no rotational motion, the weight of a person on the equation is W. Determine the speed with which the earth would have to rotate about its axis so that the person at the equator will weight 3/4 W. Radius of the earth is 6400 km and g = 10 m/s^2.

(A*) 0.63×10^{-3} rad/s (B) 0.28×10^{-3} rad/s

(C) 1.1×10^{-3} rad/s (D) 0.83×10^{-3} rad/s

Sol. $g' = g - \omega^2 R \cos^2\theta$

$\frac{3g}{4} = g\omega^2 R$

$w^2R = \frac{g}{4}$

$w = \sqrt{\frac{g}{4R}}\,4$

$= \sqrt{\frac{10}{4 \times 64\omega \times 10^3}}$

$= \frac{1}{2 \times 8 \times 100}$

$= \frac{1}{1600} = \frac{1}{16} \times 10^{-2} = 0.6 \times 10^{-3}$

38. An object is dropped from a height h from the ground. Every time it hits the ground it looses 50% of its kinetic energy. The total distance covered as $t \to \infty$ is -

(A) 2h (B) ∞ (C) $\frac{5}{3}h$ (D) $\frac{8}{3}h$

Sol. $\frac{1}{2}mv'^2 = \frac{1}{2}\frac{1}{2}mv^2$

$v' = \frac{v}{\sqrt{2}}$

v = eu

$e = \frac{1}{\sqrt{2}}$

$h = \lambda\left(\frac{1+e^2}{1-e^2}\right)$

$= h\left(\frac{1+\frac{1}{2}}{1-\frac{1}{2}}\right) = 3h$

39. The energy stored in the electric field produced by a metal sphere is 4.5 J. If the sphere contains 4μC charge, its radius will be :

[Take : $\frac{1}{4\pi\varepsilon_0} = 9 \times 10^9 N - m^2 / C^2$]

(A) 32 mm (B*) 16 mm (C) 28 mm (D) 20 mm

Sol. Energy of sphere $= \frac{Q^2}{2C}$

$4.5 = \frac{16 \times 10^{-12}}{2C}$

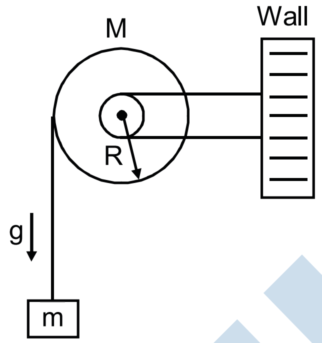
$C = \frac{16 \times 10^{-12}}{9} = 4\pi\varepsilon_0 R$

$$R = \frac{16 \times 10^{-12}}{9} \times \frac{1}{4\pi\varepsilon_0}$$

$= 9 \times 10^9 \times 16/9 \times 10^{-12}$

$= 16 \times 10^{-3} = 16$ mm

40. A uniform disc of radius R and mass M is free to rotate only about its axis. A string is wrapped over its rim and a body of mass m is tied to the free end of the string as shown in the figure. The body is released from rest. Then the acceleration of the body is -



(A) $\frac{2Mg}{2m+M}$ (B) $\frac{2Mg}{2M+m}$ (C) $\frac{2mg}{2M+m}$ (D*) $\frac{2mg}{2m+M}$

Sol. mg – T = ma

$RT = I\propto$

$$RT = \frac{MR^2}{2}.\frac{a}{R}$$

$$T = \frac{Ma}{2}$$

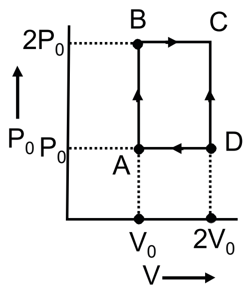
$$mg - \frac{Ma}{2} = ma$$

$$mg = a\left(\frac{M}{2} + m\right)$$

$$mg = a\left(\frac{M+2m}{2}\right)$$

$$a = \frac{2mg}{M+2m}$$

41. An engine operates by taking n moles of an ideal gas through the cycle ABCDA shown in figure. The thermal efficiency of the engine is - (Take Cv = 1.5 R, where R is gas constant)



(A) 0.24 (B*) 0.15 (C) 0.32 (D) 0.08

Sol. $w = P_0V_0$

Heat given = $Q_{AB} = Q_{BC}$

$= nC_VdT_{AB} + nC_PdT_{BC}$

$= \frac{3}{2}(nRT_B - nRT_A) + \frac{5}{2}(nRT_C - nRT_B)$

$= \frac{3}{2}(2P_0V_0 - P_0V_0) + \frac{5}{2}(4P_0V_0 - 2P_0V)$

$= \frac{13}{2}P_0V_0$

$n = \frac{w}{Qgiven} = \frac{2}{13} = 0.15$

42. Time (T), velocity (3) and angular momentum (h) are chosen as fundamental quantities instead of mass, length and time. In terms of these, the dimensions of mass would be -

(A*) $[M] = [T^{-1} C^{-2}h]$ (B) $[M] = [T C^{-2}h]$

(C) $[M] = [T^{-1} C^{-2}h^{-1}]$ (D) $[M] = [T^{-1} C^{2}h]$

Sol. $M \propto T^x v^y h^z$

$M' L^0T^0 = (T')^x (L^1T^{-1})^y (M^1L^2T^{-1})^z$

$M^1L^0T^0 = M^z L^{y+2z} + T^{x-y-z}$

$z = 1$

$y + 2z = 0$ $x - y - z = 0$

$y = -2$ $x + 2 - 1 = 0$

$x = -1$

$M \Rightarrow T^{-1} C^{-2} h^1$

43. In an experiment a sphere of aluminium of mass 0.20 kg is heated upto 150°C. Immediately, it is put into water of volume 150 cc at 27°C kept in a calorimeter of water equivalent to 0.025 kg. Final temperature of the system is 40°C. The specific heat of aluminium is -

(take 4.2 Joule = 1 calorie)

(A*) 434J/kg-°C (B) 378J/kg-°C (C) 315J/kg-°C (D) 476J/kg-°C

Sol. Q given = Q used

$0.2 \times S \times (150 - 40) = 150 \times 1 \times (40 - 27)$

$+ 25 \times (40 - 27)$

$0.2 \times S \times 110 = 150 \times 13 + 25 \times 13$

$S = \frac{13 \times 25 \times 7}{0.2 \times 110}$

$S = 434$

44. There is a uniform electrostatic field in a region. The potential at various points on a small sphere centred at P, in the region, is found to vary between in limits 589.0V to 589.8 V. What is the potential at a point on the sphere whose radius vector makes an angle of 60° with the direction of the field ?

(A*) 589.4 V (B) 589.5 V (C) 589.2 V (D) 589.6 V

Sol. $\Delta V = E.d$

$0.8 = Ed\ (max)$

$\Delta V = Ed\cos\theta = 0.8 \times \cos 60$

$= 0.4$

589.4

45. Magnetic field in a plane electromagnetic wave is given by

$\vec{B} = B_0 \sin(kx + \omega t)\hat{j}T$

Expression for corresponding electric field will be

(A) $\vec{E} = -B_0 c \sin(kx + \omega t)\hat{k} V/m$

(B) $\vec{E} = B_0 c \sin(kx - \omega t)\hat{k} V/m$

(C*) $\vec{E} = B_0 c \sin(kx + \omega t)\hat{k} V/m$

(D) $\vec{E} = \frac{B_0}{c} \sin(kx + \omega t)\hat{k} V/m$

Sol. $C = \frac{E_0}{B_0}$

$E = CB_0$

$= CB_0$

$= C_0 \sin(kx + \omega t)\ \hat{i}$

46. According to Bohr's theory, the time averaged magnetic field at the centre (i.e. nucleus) of a hydrogen atom due to the motion of electrons in the nth orbit is proportional to :

(n = principal quantum number)

(A) n^{-3} (B*) n^{-2} (C) n^{-4} (D) n^{-5}

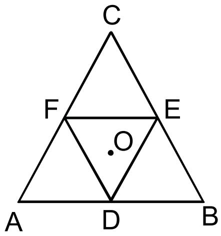
Sol. $B = \frac{\mu_0 I}{2r}$

$= \frac{\mu_0 qt}{2r}$

$r \propto n^2$

$B \propto m^{-2}$

47. Moment of inertia of an equilateral triangular lamina ABC, about the axis passing through its centre O and perpendicular to its plane is I0 as shown in the figure. A cavithy DEF is cut out from the lamina, where D,E,F are the mid points of the sides. Moment of inertia of the remaining part of lamina about the same axis is -



(A*) $\frac{15}{16}I_0$ (B) $\frac{3I_0}{4}$ (C) $\frac{7}{8}I_0$ (D) $\frac{31I_0}{32}$

Sol. $I_0 = km\ell^2$ $C = \ell$

$I_{DEF} = K\frac{m}{4}\left(\frac{\ell}{2}\right)^2$

$= \frac{k}{16}m\ell^2$

$I_{DEF} = \frac{I_0}{16}$

$I_{remain} = I_0 = \frac{I_0}{16}$

$= \frac{15I_0}{16}$

48. The maximum velocity of the photoelectrons emitted from the surface is v when light of frequency n falls on a metal surface. If the incident frequency is increased to 3n, the maximum velocity of the ejected photoelectrons will be -

(A) more than $\sqrt{3}$ v (B*) equal to $\sqrt{3}$ v (C) v (D) less than $\sqrt{3}$ v

Sol. $E_1 = hn - \phi$

$E_2 = 4hn - \phi$

$E_2 = 3(E_1 + \phi) - \phi$

$E_2 = 3E_1 + 2V$

$m_0 x\sqrt{3}$ v

49. What is the conductivity of a semiconductor sample having electron concentration of 5×10^{18} m^{-3}, hole concentration of 5×10^{19} m^{-3}, electron mobility of 2.0 $m^2 V^{-1} s^{-1}$ and hole mobility of 0.01 $m^2 V^{-1} s^{-1}$? (Take charge of electron as 1.6×10^{-19} C)

(A) 1.83 $(\Omega\text{-m})^{-1}$ (B*) 1.68 $(\Omega\text{-m})^{-1}$ (C) 1.20 $(\Omega\text{-m})^{-1}$ (D) 0.59 $(\Omega\text{-m})^{-1}$

Sol. $s = e\,(n_e\mu_e + n_n\mu_n)$

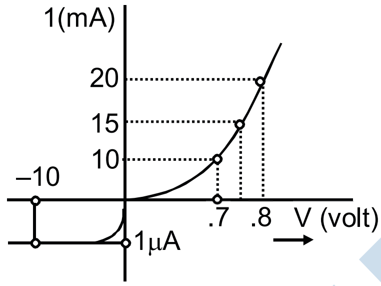
$= 1.6 \times 10^{-19}\,(5 \times 10^{18} \times 2 + 5 \times 10^{19} \times 0.01)$

$= 1.6 \times 10^{-19}\,(10^{19} + 0.05 \times 10^{19})$

$= 1.6 \times 1.05$

$= 1.68$

50. The V-I characteristic of a diode is shown in the figure. The ratio of forward to reverse bias resistance is:



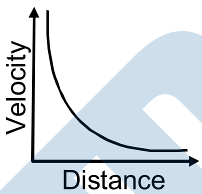
(A) 100 (B) 106 (C*) 10^{-6} (D) 10

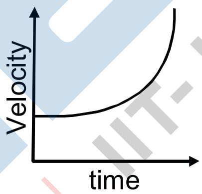
Sol. $\text{f.B} = \dfrac{0.1}{10 \times 10^{-3}} = 10$

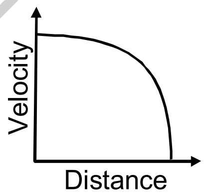
$\text{RB} = \dfrac{10}{10^{-6}} = 10^{7}$

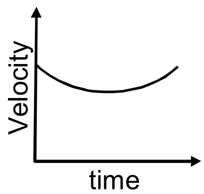
$\dfrac{\text{fB}}{\text{RB}} = 10^{-6}$

51. Which graph corresponds to an object moving with a constant negative acceleration and a positive velocity ?

(A) 

(B) 

(C) 

(D) 

Sol. $a = -C$

$\dfrac{VdV}{dx} = -C$

$VdV = -\,CdX$

$\dfrac{V^2}{2} = -Cx + k$

$x = -\dfrac{V^2}{2C} + \dfrac{K}{C}$

52. A small circular loop of wire of radius a is located at the centre of a much larger circular wire loop of radius b. The two loops are in the same plane. The outer loop of radius b carries an alternating current $I = I_0 \cos(\omega t)$. The emf induced in the smaller inner loop is nearly ?

(A) $\pi\mu_0 I_0 \frac{a^2}{b}\omega\sin(\omega t)$

(B) $\frac{\pi\mu_0 I_0}{2}\cdot\frac{a^2}{b}\omega\cos(\omega t)$

(C) $\frac{\pi\mu_0 I_0 b^2}{a}\omega\cos(\omega t)$

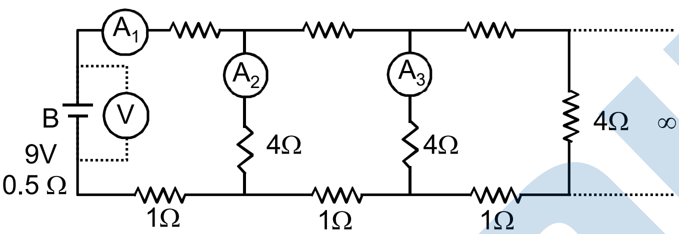
(D) $\frac{\pi\mu_0 I_0}{2}\cdot\frac{a^2}{b}\omega\sin(\omega t)$

Sol. e = MdI

dT $\qquad M = \frac{\mu_0 \pi a^2}{2b}$

$= \frac{\mu_0 \pi a^2}{2b} = \omega I_0 \; l\omega t$

53.



A 9 V battery with internal resistance of 0.5 Ω is connected across an infinite network as shown in the figure. All ammeters A_1, A_2, A_3 and voltmeter V are ideal.

Choose correct statement.

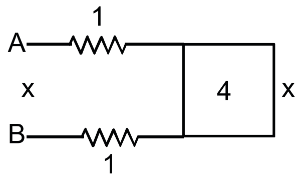
(A) Reading of A_1 is 18 A

(B) Reading of V is 9 V

(C) Reading of V is 7 V

(D*) Reading of A_1 is 2 A

Sol. $x = \frac{4x}{4+x} + 2$



$x = \frac{8+6x}{4+x}$

$4x + x^2 = 8 + 6x$

$x^2 - 2x - 8 = 0$

$x = \frac{2 \pm \sqrt{4-4(1)(-8)}}{2} = \frac{2 \pm \sqrt{36}}{2}$

$x = \frac{2 \pm 6}{2} = 4$

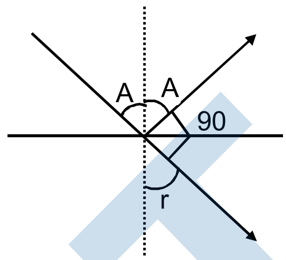
$A' = \frac{9}{4+0.5} = 2$

A' = 9

54. Let the refractive index of a denser medium with respect to a rarer medium be n_{12} and its critical angle be θ_C. At an angle of incidence A when light is travelling from denser medium to rarer medium, a part of the light is reflected and the rest is refracted and the angle between reflected and refracted rays is 90°. Angle A given by -

(A*) $\tan^{-1}(\sin\theta_C)$ (B) $\frac{1}{\tan^{-1}(\sin\theta_C)}$ (C) $\cos^{-1}(\sin\theta_C)$ (D) $\frac{1}{\cos^{-1}(\sin\theta_C)}$

Sol. $\mu = \frac{\mu_R}{\mu_D} = \frac{\sin i_C}{\sin 90°}$



$\frac{\mu_R}{\mu_D} = \sin i_i$

$\mu = \frac{\mu_R}{\mu_D} = \frac{\sin A}{\sin r}$

$= \frac{\sin A}{\sin(90 - A)} = \frac{\sin A}{\cos A}$

$\frac{\mu_R}{\mu_D} = \tan A$

$\tan A = \sin\theta_C$

$A = \tan^{-1}(\sin\theta_C)$

55. The ratio of maximum acceleration to maximum velocity in a simple harmonic motion is 10 s^{-1}. At, t = 0 the displacement is 5 m. What is the maximum acceleration ? The initial phase is $\frac{\pi}{4}$.

(A) 500 m/s^2 (B) $750\sqrt{2}\ m/s^2$ (C) 750 m/s^2 (D*) $500\sqrt{2}\ m/s^2$

Sol. $f_{max} = \omega a$

$v_{min} = a\omega$

$\frac{\omega a}{a\omega} = 10$

w = 10

$x = a\sin(\omega + \pi/4)$

at f = 0

$5 = a\sin(\pi/4)$

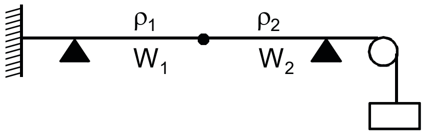
$a = 5\sqrt{2}$

max acc. = w2a

$= 100 \times 5\sqrt{2}$

$= 500\sqrt{2}$

56. Two wires W_1 and W_2 have the same radius r and respective densities ρ_1 and ρ_2 such that $\rho_2 = 4\rho_1$. They are joined together at the point O, as shown in the figure. The combination is used as a sonometer wire and kept under tension T. The point O is midway between the two bridges. When a stationary waves is set up in the composite wire, the joint is found to be a node. The ratio of the number of antinodes formed in W_1 to W_2 is -



(A) 4 : 1 (B*) 1 : 2 (C) 1 : 1 (D) 1 : 3

Sol. $n_1 = n_2$

$T \rightarrow$ same

$r \rightarrow$ same

$\ell \rightarrow$ same

$$n = \frac{p}{2\ell}\sqrt{\frac{T}{\pi r^2 d}}$$

$n_1 = n_2$

$$\frac{p_1}{\sqrt{d_1}} = \frac{p_2}{\sqrt{d_2}}$$

$$\frac{p_1}{p_2} = \frac{1}{2}$$

57. An ideal gas has molecules with 5 degrees of freedom. The ratio of specific heats at constant pressure (C_P) and at constant volume (C_V) is

(A*) $\frac{7}{5}$ (B) 6 (C) $\frac{7}{2}$ (D) $\frac{5}{2}$

Sol. $f = \frac{C_p}{C_a} = 1 + \frac{2}{1}$

= 7/5

58. Two deuterons udnergo nuclear fusion to form a Helium nucleus. Energy released in this process is : (given binding energy per nucleon for deuteron = 1.1 MeV and for helium = 7.0 MeV)

(A*) 23.6 MeV (B) 25.8 MeV (C) 30.2 MeV (D) 32.4 MeV

Sol. ${}_1H^2 + {}_1H^1 \longrightarrow 2H_c^4$

initiate $\Rightarrow$ 1.1 × 4 = 4.4

final $\Rightarrow$ 4 × 7 = 28

release $\Rightarrow$ 28 – 4.4 = 23.6

59. In a certain region static electric and magnetic fields exist. The magnetic field is given by $\vec{B} = B_0\left(\hat{i} + 2\hat{j} - 4\hat{k}\right)$. If a test charge moving with a velocity $\vec{v} = v_0\left(3\hat{i} - \hat{j} + 2\hat{k}\right)$ experience no force in that region, then the electric field in the region, in SI units, is -

(A) $\vec{E} = -v_0B_0\left(\hat{i} + \hat{j} + 7\hat{k}\right)$ (B) $\vec{E} = -v_0B_0\left(3\hat{i} - 2\hat{j} - 4\hat{k}\right)$

(C) $\vec{E} = v_0B_0\left(14\hat{j} + 7\hat{k}\right)$ (D*) $\vec{E} = -v_0B_0\left(14\hat{j} + 7\hat{k}\right)$

Sol. $F_e = F_n = 0$

$F_e = -F_m$

$= -q\left(\vec{v} \times \vec{B}\right)$

$= -v_0v_0\left|\left(3\hat{i} - \hat{j} + 2\hat{k}\right) \times \left(\hat{i} + 2\hat{j} - 4\hat{k}\right)\right|$

$= -v_0v_0\left(14\hat{j} + 7\hat{k}\right)$

60. In a physical balance working on the principle of moments, when 5 mg weight is placed on the left pan, the beam becomes horizontal. Both the empty pans of the balance are of equal mass. Which of the following statements is correct ?

(A) Every object that is weighted using this balance appears lighter than its actual weight

(B*) Left arm is shorter than the right arm

(C) Both the arms are of same length

(D) Left arm is longer than the right arm

PART – C – CHEMISTRY

61. Among the following, correct statement is:

(A) Brownian movement is more pronounced for smaller particles than for bigger–particles.

(B) Sols of metal sulphides are lyophilic.

(C) Hardy Schulze law states that bigger the size of the ions, the greater is its coagulating power.

(D*) One would expect charcoal to adsorb chlorine more than hydrogen sulphide.

62. Excess of NaOH (aq) was added to 100 mL of $FeCl_3$ (aq) resulting into 2.14 g of $Fe(OH)_3$. The molarity of $FeCl_3$ (aq) is:

[Given molar mass of Fe=56 g mol^{-1} and molar mass of Cl=35.5 g mol^{-1}]

(A*) 0.2 M (B) 0.3 M (C) 0.6 M (D) 1.8 M

Sol. $3NaOH + FeCl_3 \longrightarrow Fe(CH)_3 + NaCl$

100 ml 2.14 gm

m = ?

Moles of $Fe(CH_3) = \frac{2.14}{107} = 2 \times 10^{-2} mol$

moles $FeCl_3 = 2 \times 10^{-2}$ mol

$$M = \frac{2 \times 10^{-2}}{100} \times 1000 = 0.2\ M$$

63. Among the following, the incorrect statement is :

(A) At low pressure, real gases show ideal behaviour.

(B*) At very low temperature, real gases show ideal behaviour.

(C) At very large volume, real gases show ideal behaviour.

(D) At Boyle's temperature, real gases show ideal behaviour.

64. For a reaction, $A(g) \rightarrow A(l)$; $\Delta H = -3RT$.

The correct statement for the reaction is:

(A) $\Delta H = \Delta U \neq 0$ (B) $\Delta H = \Delta U = 0$ (C*) $|\Delta H| < |\Delta U|$ (D) $|\Delta H| > |\Delta U$

65. What is the standard reduction potential (E°) for $Fe^{3+} \rightarrow Fe$?

Given that:

$Fe^{2+} + 2e^- \rightarrow Fe$; $E^o_{Fe^{2+}/Fe} = -0.47\,V$

$Fe^{3+} + e^- \rightarrow Fe^{2+}$; $E^o_{Fe^{3+}/Fe^{2+}} = +0.77\,V$

(A*) – 0.057 V (B) +0.057 V (C) +0.30 V (D) .0.30 V

66. If the shortest wavelength in Lyman series of hydrogen atom is A, then the longest wavelength in Paschen series of He^+ is:

(A) $\frac{5A}{9}$ (B) $\frac{9A}{5}$ (C) $\frac{36A}{5}$ (D*) $\frac{36A}{7}$

Sol. Shortest wavelength is corresponding to best ine

$\therefore$ $n_L = 1$ (Lyman series)

$n_H = \infty$ (infinite)

$$\frac{1}{A} = r \times (1)^2 \left\{\frac{1}{12} - \frac{1}{2}\right\} = R$$

Longest wavelength $\equiv 1^{st}$ Line

$\therefore = 3$ $n_H = 4$

$$\frac{1}{\lambda} = r \times (2)^2 \left\{\frac{1}{3^2} - \frac{1}{4^2}\right\} = \frac{r \times 7}{36}$$

$$\lambda = \frac{36A}{7}$$

67. 5 g of Na_2SO_4 was dissolved in x g of H_2O. The change in freezing point was found to be 3.82°C. If Na_2SO_4 is 81.5% ionised, the value of x

(K_f for water =1.86°C kg mol^{-1}) is approximately:

(molar mass of S=32 g mol^{-1} and that of Na=23 g mol^{-1})

(A) 15 g (B) 25 g (C*) 45 g (D) 65 g

Sol. $Na_2SO_4 \longrightarrow 2Na^+ + SO_4^{2-}$

x = 1 + (3 – 1) 0.815 = 2.63

$$3.82 = 1.86 \times 2.63 \times \frac{5 \times 1000}{142 \times x}$$

$$\therefore x = \frac{1.86 \times 2.63 \times 5000}{142 \times 3.82}$$

= 45 gm

68. Addition of sodium hydroxide solution to a weak acid (HA) results in a buffer of pH 6. If ionisation constant of HA is 10^{-5}, the ratio of salt to acid concentration in the buffer solution will be:

(A) 4: 5 (B) 1: 10 (C*) 10: 1 (D) 5: 4

69. The rate of a reaction A doubles on increasing the temperature from 300 to 310 K. By how much, the temperature of reaction B should be increased from 300 K so that rate doubles if activation energy of the reaction B is twice to that of reaction A.

(A) 9.84 K (B*) 4.92 K (C) 2.45 K (D) 19.67 K

Sol. $2 = \frac{Eq}{R}\left\{\frac{1}{300} - \frac{1}{310}\right\}$ ...(i)

$2 = e^2 \frac{Ea}{R}\left\{\frac{1}{300} - \frac{1}{T}\right\}$ ...(ii)

$$\frac{2Ea}{R}\left\{\frac{1}{300} - \frac{1}{T}\right\} = \frac{E_a}{R}\left\{\frac{1}{300} - \frac{1}{310}\right\}$$

$$\frac{1}{300}+\frac{1}{310}=\frac{2}{T}\Rightarrow T=\frac{300\times 310}{610}\times 2$$

= 304.92

70. The enthalpy change on freezing of 1 mol of water at 5°C to ice at –5°C is:

(Given ΔH_{fus}=6 kJ mol^{-1} at 0°C, $C_p(H_2O, \ell)$=75.3 J mol^{-1} K^{-1}, $C_p(H_2O, s)$=36.8 J mol^{-} K^{-1})

(A) 5.44 kJ mol^{-1} (B) 5.81 kJ mol^{-1} (C*) 6.56 kJ mol^{-1} (D) 6.00 kJ mol^{-1}

71. Which of the following is paramagnetic?

(A) NO^+ (B) CO (C) O_2^{2-} (D*) B_2

Sol. No of e–

CO = 14, NO^+ = 14

$O_2^{2+}=18$ $B_2=10$

According to MOT

B_2 is paramagnetic

72. The pair of compounds having metals in their highest oxidation state is:

(A*) MnO_2 and CrO_2Cl_2 (B) $[NiCl_4]^{2-}$ and $[CoCl_4]^{2-}$

(C) $[Fe(CN)_6]^{3-}$ and $[Cu(CN)_4]^{2-}$ (D) $[FeCl_4]^-$ and Co_2O_3

Sol. MnO_2 = + 4

CrO_2Cl_2 = + 6

73. sp^3d^2 hybridization is not displayed by:

(A) BrF_5 (B) SF_6 (C) $[CrF_6]^{3-}$ (D*) PF_5

Sol. $SF_6 = Sp^3d^2$ $BrF_5 = SP^3d^2$

$[CrF_6]^{3-} = sp^3d^2$ PFs = sp^3d

74. Identify the pollutant gases largely responsible for the discoloured and lusterless nature of marble of the Taj Mahal.

(A) O_3 and CO_2 (B) CO_2 and NO_2 (C*) SO_2 and NO_2 (D) SO_2 and O_3

Sol. SO_2 and NO_2

75. In which of the following reactions, hydrogen peroxide acts as an oxidizing agent?

(A) $HOCl + H_2O_2 \longrightarrow H_3O^+ + Cl^- + O_2$

(B) $I_2 + H_2O_2 + 2OH^- \longrightarrow 2I^- + 2H_2O + O_2$

(C) $2MnO_4^- + 3H_2O_2 \longrightarrow 2MnO_2 + 3O_2 + 2H_2O + 2OH^-$

(D*) $PbS + 4H_2O_2 \longrightarrow PbSO_4 + 4H_2O$

Sol. $PbS + 4H_2O_2 \longrightarrow PbSO_4 + 4H_2O$

+2 +4

76. Consider the following ionization enthalpies of two elements 'A' and 'B'.

Element	Ionisation enthalpy (kJ/mol)		
	1^{st}	2^{nd}	3^{rd}
A	899	1757	14847
B	737	1450	7731

Which of the following statements is correct?

(A) Both 'A' and 'B' belong to group-1 where 'B' comes below 'A'.

(B) Both 'A' and 'B' belong to group-1 where 'A' comes below 'B'.

(C*) Both 'A' and 'B' belong to group-2 where 'B' comes below 'A'.

(D) Both 'A' and 'B' belong to group-2 where 'A' comes below 'B'.

77. Consider the following standard electrode potentials (E° in volts) in aqueous solution:

Element	M^{3+} / M	M^{+} / M
Al	–1.66	+0.55
Tl	+1.26	–0.34

Based on these data, which of the following statements is correct

(A) Tl^{+} is more stable than Al^{3+}

(B) Al^{+} is more stable than Al^{3+}

(C*) Tl^{+} is more stable than Al^{+}

(D) Tl^{3+} is more stable than Al^{3+}

Sol. ΔG is –ve

78. A metal 'M' reacts with nitrogen gas to afford 'M_3N'. 'M_3N' on heating at high temperature gives back 'M' and on reaction with water produces a gas 'B'. Gas 'B' reacts with aqueous solution of $CuSO_4$ to form a deep blue compound. 'M' and 'B' respectively are:

(A*) Li and NH_3 (B) Ba and N_2 (C) Na and NH_3 (D) Al and N_2

Sol. $Li + N_2 \longrightarrow Li_3N$

$\downarrow H_2O$

$LiOH + NH_3$

$\downarrow CoSO_4$

$[Cu(NH_3)_4]\,SO_4$

79. The number of S = O and S – OH bonds present in peroxodisulphuric acid and pyrosulphuric acid respectively are:

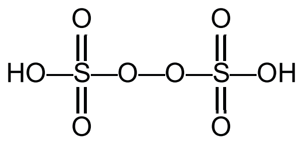
(A) (2 and 2) and (2 and 2)

(B) (2 and 4) and (2 and 4)

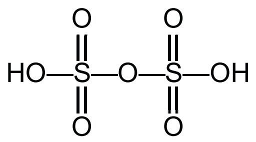
(C) (4 and 2) and (2 and 4)

(D*) (4 and 2) and (4 and 2)

Sol. Peroxodisulphuric acid

$H_2S_2O_8$ 

Pyrosulphuric acid

$H_2S_2O_7$: 

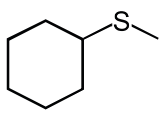
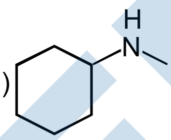
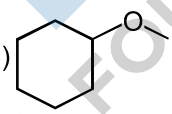
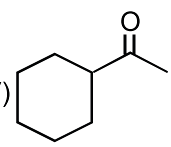
80. A solution containing a group-IV cation gives a precipitate on passing H_2S. A solution of this precipitate in dil.HCl produces a white precipitate with NaOH solution and bluish-white precipitate with basic potassium ferrocyanide. The cation is:

(A) Co^{2+} (B) Ni^{2+} (C) Mn^{2+} (D*) Zn^{2+}

Sol. $ZnS \xrightarrow{HCl} ZnCl_2 \xrightarrow{NaOH} Zn(OH)_2$

$ZnCl_2 \xrightarrow{K_4[Fe(CN)_6]} Zn_2[Fe(CN)_6]$

81. A mixture containing the following four compounds is extracted with 1M HCl. The compound that goes to aqueous layer is:

(I)  (II)  (III)  (IV) 

(A) (I) (B*) (II) (C) (III) (D) (IV)

82. The reason for "drug induced poisoning" is:

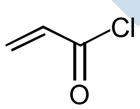
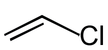
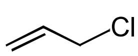
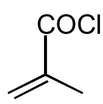
(A) Binding reversibly at the active site of the enzyme

(B) Bringing conformational change in the binding site of enzyme

(C) Binding irreversibly to the active site of the enzyme

(D*) Binding at the allosteric sites of the enzyme

83. Which of the following compounds will not undergo Friedel Craft's reaction with benzene?

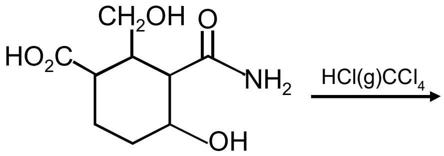
(A)  (B*)  (C)  (D) 

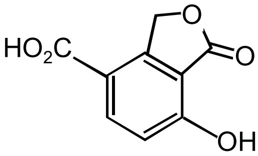
Sol. Formation of carbocation is not possible in case of $CH_2 = CHCl$

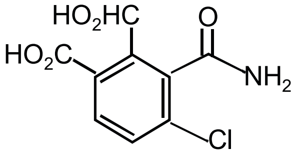
84. Among the following, the essential amino acid is:

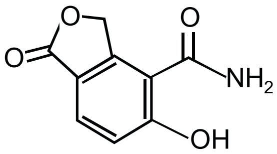
(A) Alanine (B*) Valine (C) Aspartic acid (D) Serine

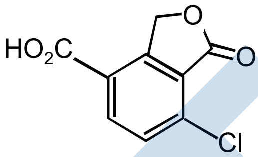
85. The major product expected from the following reaction is:

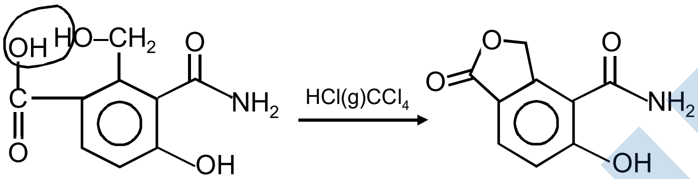


(A) 

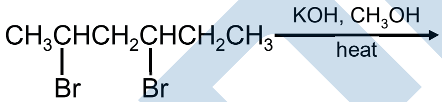
(B) 

(C*) 

(D) 

Sol. 

86. The major product of the following reaction is:

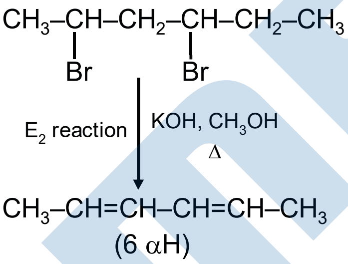


(A) $CH_2 = CHCH_2CH = CHCH_3$

(B) $CH_2 = CHCH = CHCH_2CH_3$

(C) $CH_3CH = C = CHCH_2CH_3$

(D*) $CH_3CH = CH – CH = CHCH_3$

Sol. 

87. Which of the following statements is not true about partition chromatography?

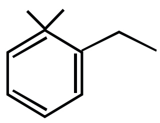
(A*) Mobile phase can be a gas

(B) Stationary phase is a finely divided solid adsorbent

(C) Separation depends upon equilibration of solute between a mobile and a stationary phase

(D) Paper chromatography is an example of partition chromatography

88. The IUPAC name of the following compound is:

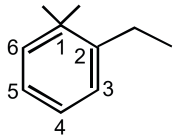


(A) 1, 1-Dimethyl-2-ethylcyclohexane

(B*) 2-Ethyl-1, 1-dimethylcyclohexane

(C) 1-Ethyl-2, 2-dimethylcyclohexane

(D) 2, 2-Dimethyl-1-ethylcyclohexane

Sol. 

89. The major product of the following reaction is:

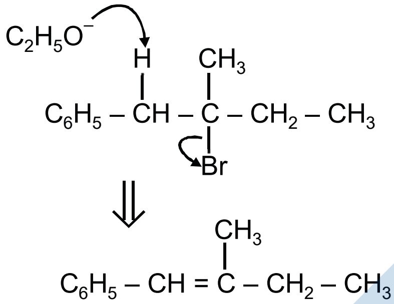
$$C_6H_5CH_2-\underset{Br}{\overset{CH_3}{C}}-CH_2-CH_3 \xrightarrow[C_2H_5OH]{C_2H_5ONa}$$

(A) $C_6H_5CH_2-\underset{OC_2H_5}{\overset{CH_3}{C}}-CH_2CH_3$

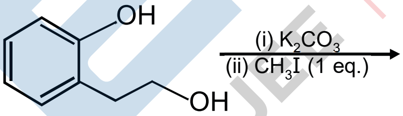
(B*) $C_6H_5CH=\underset{CH_3}{C}-CH_2CH_3$

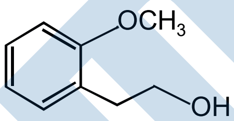
(C) $C_6H_5CH_2-\underset{CH_3}{C}=CHCH_3$

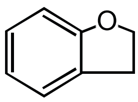
(D) $C_6H_5CH_2-\underset{CH_2CH_3}{C}=CH_2$

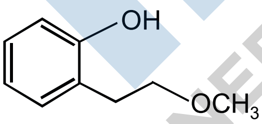
Sol. 

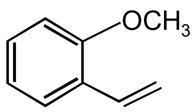
90. The major product of the following reaction is:



(A*) 

(B) 

(C) 

(D) 

Sol.